EFFECT OF DROPLET SIZE DISTRIBUTION ON SPRAY DRYING KINETICS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 1, pp. 36–43, 1968

UDC 66.047

A comparison is made between the processes of adiabatic evaporation of a polydisperse system of droplets and a monodisperse system with droplets equal in size to the largest droplet of the polydisperse system. The process takes place in air with the same initial and final parameters in both cases.

The atomizers used in spray drying apparatus disperse the liquid into droplets of various size, with droplet diameters differing in some cases by a factor of 8–10.

The volume and height of the drying chamber are determined by the trajectory and length of the path followed by the largest droplet during the drying period. If for a given output and drying regime the volume of the chamber is less than that required for the largest droplets, incompletely dried product will be deposited on the walls of the chamber.

In the calculation of the chamber volume the polydisperse droplet system is replaced by a reduced monodisperse system, whose over-all volume and droplet surface are equal to those of the polydisperse system [1,4]. The droplet size in the reduced system is given by

$$d_{3,2}=\frac{\sum\limits_{d_{\min}}^{d_{\max}} d_i^3 r_i}{\sum\limits_{d_{\min}}^{d_{\max}} d_i^2 r_i}.$$

The volume of the drying chamber is found from the mass transfer equation [9]

$$V=\frac{W}{K_V\Delta P}. \tag{1}$$

The fact that the same $d_{3,2}$ for the reduced monodisperse system corresponds to polydisperse systems with very different fractional compositions is disregarded. This is because in modern equipment the atomized liquid volume distribution with respect to droplet size almost corresponds with the relation $\Omega = \exp(-d/d_e)$, the value of n varying in the narrow range 2–4 [2].

The conditions of evaporation are essentially different for the polydisperse and monodisperse systems. The evaporation time, under the same initial and final conditions, is different for a polydisperse system of droplets whose largest diameter is d_{in}, and a monodisperse system of equal droplets of diameter d_{in}.

In a polydisperse system the presence of a developed phase contact surface during the initial period intensifies the mass transfer process (as compared with the monodisperse system). During this period most of the very fine droplets evaporate, the humidity of the air increases sharply, and the motive force of the process is correspondingly reduced. The final evaporation of the large droplets takes place under a reduced motive force. This leads to an increase in the time required to complete the process as compared with the evaporation time for the monodisperse system, in which the motive force is less sharply reduced.

To verify this picture, we calculate the corresponding evaporation times for monodisperse and polydisperse systems of water droplets in moist air under the same initial and final conditions. We assume that the droplet size in the monodisperse system is equal to the maximum* droplet size in the polydisperse system. To obtain generalized results, we express the parameters in relative quantities.

The evaporation time for a droplet of initial diameter d_{in} and final diameter $\xi_m d_{in}$ in air whose parameters do not vary during the process is [2, 3, 5–8].

$$\tau_m=\frac{\gamma d_{\text{in}}^2(1-\xi_m^2)}{4\text{Nu}\,K\Delta P}, \tag{2}$$

where

$$K=D_c^0\left(\frac{T}{273}\right)^{3/2}\frac{M}{RT},\qquad K=D_c^0\frac{T^{1/2}M}{273^{3/2}R}$$

for ΔP = const, Nu = const, T = const (isothermal evaporation process in an unbounded space) Eq. (2) may be rewritten as follows:

$$\xi_m^2 d_{\text{in}}^2=d_{\text{in}}^2-N\tau_m, \tag{3}$$

where

$$N=\frac{4\text{Nu}\,K\Delta P}{\gamma}=\text{const}.$$

At a time τ_m from the beginning of the process, a droplet of initial diameter $\xi_i d_{in}$ evaporating under the same conditions, will have a diameter

$$\xi_{im}^2 d_{\text{in}}^2=\xi_i^2 d_{\text{in}}^2-N\tau_m. \tag{4}$$

Relations (3) and (4) were established by B. V. Sreznevskii [5] and have been confirmed by a number of investigators.

In parallel-flow drying chambers the evaporation process is adiabatic, the temperature and humidity of the drying agent varying during the process.

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*As compared with the mean diameter the difference will clearly be greater.

Since under industrial conditions the mean boundary layer temperature of the evaporating droplets varies only slightly, it has little effect on the mass transfer coefficient. Therefore, with sufficient accuracy, we assume that

$$K = D_c^0 \frac{\left[\dfrac{T_{\text{м}} + 1/2(T_{\text{in}} + T_{\text{f}})^{1/2}}{2}\right]}{273^{3/2} R} = \text{const.}$$

The following material balance equations can be written for the moment of total evaporation of the droplet and its evaporation to the diameter $d_{\text{in}}\xi_{\text{m}}$

$$L(X_{\text{f}} - X_{\text{in}}) = \frac{\pi d_{\text{in}}^3}{6}\gamma,$$

$$L(X_m - X_{\text{in}}) = \frac{\pi d_{\text{in}}^3}{6}\gamma(1 - \xi_m^3),$$

whence there follows

$$X_m - X_{\text{in}} = (X_{\text{f}} - X_{\text{in}})(1 - \xi_m^3)$$

$$\text{and } X_m = X_{\text{in}} + (X_{\text{f}} - X_{\text{in}})(1 - \xi_m^3).$$

The motive force of the process at any time, and in the range of diameter variation from d_{in} to $\xi_{\text{m}} d_{\text{in}}$ will be

$$\Delta X_m = X_{\text{м}} - X_m =$$

$$= X_{\text{м}} - X_{\text{in}} - (X_{\text{f}} - X_{\text{in}})(1 - \xi_m^3) =$$

$$= \Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}})(1 - \xi_m^3)$$

or

$$\Delta P_m = \frac{BX_{\text{м}}}{0.622 + X_{\text{м}}} - \frac{BX_m}{0.622 + X_m} =$$

$$= \frac{0.622B}{0.622 + X_{\text{м}}} \frac{\Delta X_m}{0.622 + X_m} = \frac{0.622B}{0.622 + X_{\text{м}}} \times$$

$$\times \frac{\Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}})(1 - \xi_m^3)}{0.622 + X_{\text{in}} + \Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}})(1 - \xi_m^3)}.$$

If a polydisperse system of droplets whose diameter is defined as a fraction of the diameter of the largest droplet $\xi_{\text{i}} = d_{\text{i}}/d_{\text{in}}$ is evaporated in air with fixed parameters, and the number of droplets of diameter $d_{\text{in}}\xi_{\text{i}}$ is

$$r_i = \frac{r_{\text{in}} \Delta \Omega_i}{\xi_i^3 \Delta \Omega_{\text{in}}},$$

then, when the diameter of the largest droplet is a fraction ξ_{m} of the initial diameter, on the basis of a joint solution of (3) and (4) the diameters of all the other droplets will have decreased to the value

$$\xi_{im} = [\xi_i^2 - (1 - \xi_m^2)]^{1/2},$$

and the volume of evaporated liquid as a fraction of the over-all droplet volume will be

$$A_m^n = 1 - \sum_{\xi_i=0}^{\xi_i=1} [\xi_i^2 - (1 - \xi_m^2)]^{3/2} r_i.$$

It has been assumed that the total evaporation time for the largest droplet determines the time required for the complete evaporation of the polydisperse system. In order to calculate approximately the time of total adiabatic evaporation for the largest droplet of a polydisperse system in a bounded volume of air the duration of the process is divided into m intervals, each bounded by two intermediate values of ξ_{m}. On each interval the actual process is arbitrarily replaced by a process of evaporation at fixed air parameters corresponding to the averaged values of the same parameters for the actual process. Then, using Eq. (2), it is possible to determine and sum the evaporation times for each stage of the process.

The volume of evaporated liquid as a fraction of the over-all droplet volume on the interval bounded by the values ξ_{m} and $\xi_{\text{m+1}}$ can be determined from

$$A_{m-(m+1)}^n = \frac{1}{2}(A_m^n + A_{m+1}^n) = 1 - \frac{1}{2}\Bigg\{\sum_{\xi_i=0}^{\xi_i=1} [\xi_i^2 -$$

$$- (1 - \xi_m^2)]^{3/2} r_i + \sum_{\xi_i=0}^{\xi_i=1} [\xi_i^2 - (1 - \xi_{m+1})]^{3/2} r_i\Bigg\}.$$

For this case the motive force is

$$\Delta P_{m-(m+1)} = \frac{0.622B}{0.622 + X_{\text{м}}} \times$$

$$\times \frac{[\Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}}) A_{m-(m+1)}^n]}{0.622 + X_{\text{in}} + \Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}}) A_{m-(m+1)}^n}. \tag{4'}$$

Using expressions (2), (3), and (4'), we find the total evaporation time for the polydisperse system

$$\tau_{\text{P}} = \frac{\pi d_{\text{in}}^2 (0.622 + X_{\text{м}})}{4\,\text{Nu}\,K \cdot 0.622} \times$$

$$\times \sum_{\xi_m=1}^{\xi_m=0} \frac{0.622 + X_{\text{in}} + \Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}}) A_{m-(m+1)}^n}{\Delta X_{\text{in}} - (\Delta X_{\text{in}} - \Delta X_{\text{f}}) A_{m-(m+1)}^n} \times$$

$$\times (\xi_m^2 - \xi_{m+1}^2). \tag{5}$$

Since for the monodisperse system all the droplets evaporate in the same time, the calculations can be based on the interaction between a single droplet and the corresponding unit quantity of air.

The evaporation time for a droplet with initial diameter d_{in} can be determined with the same method as used for the polydisperse system. In this case instead of $A^{\text{n}}_{\text{m}-(\text{m}+1)}$ we must substitute into (5) the quantity

$$A_{m-(m+1)}^{\text{м}} = 1 - \frac{\xi_m^3 + \xi_{m+1}^3}{2}.$$

We can now compare the evaporation time for a polydisperse system of droplets, the largest of which has the diameter d_{in}, with the corresponding time for a single droplet with diameter d_{in}, under the same initial and final air parameters

$$\frac{\tau_{\text{P}}}{\tau_{\text{м}}} = \sum_{\xi_m=1}^{\xi_m=0} \Bigg\{\Bigg[(0.622 + X_{\text{in}}) \times$$

$$\times \frac{1}{\Delta X_{\text{in}}[1-(1-\left(1-\frac{\Delta X_{\text{f}}}{\Delta X_{\text{in}}}\right)A^{n}_{m-(m+1)}]}+1\Bigg]\Bigg/$$

$$\Bigg/\left[\frac{0.622+X_m}{\Delta X_{\text{in}}[1-\left(1-\frac{\Delta X_{\text{f}}}{\Delta X_{\text{in}}}\right)A^{\text{M}}_{m-(m+1)}]}+1\right]\Bigg\}. \quad (6)$$

Equation (6) was used to calculate τ_{m} for a polydisperse system whose continuous distribution is given by

$$\Omega = \exp\left(-\frac{d}{d_e}\right)^3,$$

where $d/d_{\text{e}} = 1.8$, and the adiabatic evaporation process is characterized by $X_{\text{in}} = 0.01$ kg/kg dry air. Based on our results, we obtained the graph shown in Fig. 1, from which it follows that, given the same conditions, the evaporation time for a polydisperse system is always greater than that for a monodisperse system of droplets whose dimensions are equal to the size of the largest droplet of the polydisperse system, the difference increasing as $\Delta P_{\text{f}}/\Delta P_{\text{in}}$ decreases.

The calculated relation between $\Delta P_{\text{f}}/\Delta P_{\text{in}}$ and $d_{\text{m}}/d_{\text{in}}$ is presented in Fig. 2. This graph corresponds to the processes that take place in actual parallel-flow drying chambers, when the drier temperature falls sharply in the atomization zone and then diminishes only slightly over the rest of the chamber.

In the calculations we employed the "mean" volume distribution over the particle sizes of the polydisperse system, and consequently $d_{\text{in}}/d_{3,2}$ was assumed constant.

In this case, using the method described above, it is easy to establish a relation between the evaporation times of a polydisperse system and a monodisperse system consisting of droplets of diameter $d_{3,2}$, on the basis of which it is possible to determine the necessary correction for the mean value of the motive force in Eq. (1).

The need for this correction becomes apparent after an inspection of Fig. 1, from which it follows that $\tau_{\text{P}}/\tau_{\text{M}}$ reaches a considerable value under "severe" drying conditions.

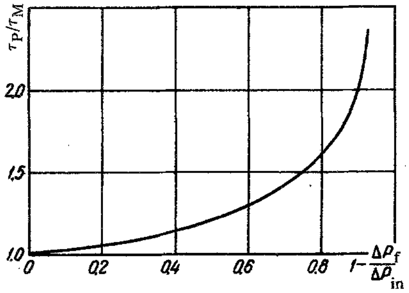

Fig. 1. Increase in the evaporation time of a polydisperse system of droplets as compared with the evaporation time of a monodisperse system as a function of the ratio of the motive forces at the beginning and end of the process.

The calculations are valid for the evaporation of monodisperse and polydisperse systems of water droplets.

The data obtained can also be used in designing parallel-flow chambers for spray drying essentially liquid products with a low concentration of dry material, for which the period of constant drying rate predominates. There is reason to assume that during the period with decreasing drying rate the nature of the effect of polydispersity on the heat and mass transfer processes remains the same.

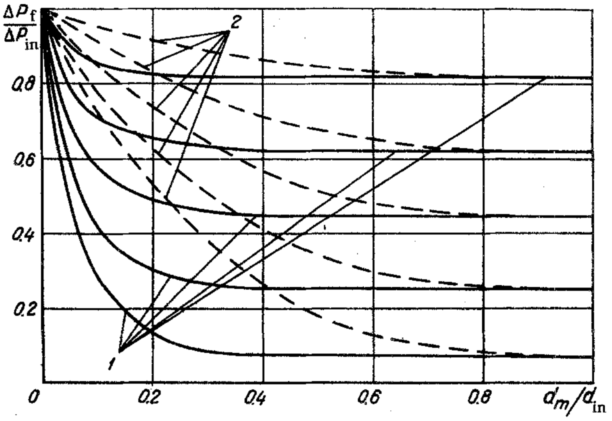

Fig. 2. Change of motive force as a function of the variation of the diameter of the largest droplet: 1) polydisperse system; 2) monodisperse system.

The method described can also be applied to other mass and heat transfer processes between continuous and dispersed media.

NOTATION

$A^{\text{M}}_{\text{m}-(\text{m}+1)}$ is the weight of the liquid as a fraction of the initial over-all weight of the droplets in a monodisperse system evaporating on an interval bounded by the droplet dimensions $\xi_{\text{m}}d_{\text{in}}$ and $\xi_{\text{m}+1}d_{\text{in}}$; A^{n}_{m} is the weight of the evaporated liquid as a fraction of the initial over-all weight of the droplets in a polydisperse system, corresponding to the diameter $\xi_{\text{m}}d_{\text{in}}$ of the largest droplet; $A^{\text{n}}_{\text{m}-(\text{m}+1)}$ is the weight of the liquid as a fraction of the initial over-all weight of the droplets in a polydisperse system evaporating on an interval bounded by the dimensions $\xi_{\text{m}}d_{\text{in}}$ and $\xi_{\text{m}+1}d_{\text{in}}$ of the largest droplet; B is the barometric pressure, mm Hg; d_{e} is the droplet diameter for which $\Omega = e^{-1}$, m; $d_{3,2}$ is the mean volume-surface diameter of a droplet, m; d_{in} is the initial diameter of the largest droplet in the polydisperse system, m; d_{i} is the discrete droplet diameter, m; D^0_{c} is the diffusion coefficient, m^2/hr; K is the mass transfer coefficient for the vapor-gas mixture, kg/m · hr · mm Hg; K_{V} is the volume mass transfer coefficient, kg/m^3 · · hr · mm Hg; L is the weight of the dry part of air, kg; M is the molecular weight of the liquid, kg/mole; Nu is the Nusselt number; ΔP is the mean value of the motive force of the process—the mean difference between the partial pressure of the saturated vapor on the droplet surface at the wet-bulb temperature and the partial vapor pressure in the air, mm Hg;

ΔP_m is the motive force of the process corresponding to the diameter $\xi_m d_{in}$ of the evaporating droplet, mm Hg; $\Delta P_{m-(m+1)}$ is the mean value of the motive force on an interval bounded by the dimensions ξd_{in} and $\xi_{m+1} d_{in}$ of the evaporating droplet, mm Hg; V is the volume of the drying chamber, m^3; R is the gas constant, mm Hg/mole · deg; r_{in} is the number of droplets of diameter d_{in}; r_i is the number of droplets of diameter d_i; T is the mean boundary layer temperature during the process °K; T_M is the wet-bulb temperature, °K; T_{in} is the initial air temperature, °K; T_f is the air temperature at the end of the process, °K; X_{in} is the initial moisture content of the air, kg/kg dry air; X_m is the moisture content of the air, corresponding to the diameter $\xi_m d_{in}$ of the evaporating droplet, kg/kg dry air; X_f is the final moisture content of the air, kg/kg dry air; X_M is the moisture content of the saturated air at the wet-bulb temperature, kg/kg dry air; ΔX_{in} is the initial difference between the moisture content of the air at the droplet surface and in the surrounding space, kg/kg dry air; ΔX_m is the difference between the moisture content of the air at the droplet surface and in the surrounding space corresponding to the diameter $\xi_m d_{in}$ of the evaporating droplet, kg/kg dry air; ΔX_f is the final difference between the moisture content of the air at the droplet surface and in the surrounding space, kg/kg dry air; W is the amount of moisture evaporated per unit time, kg/hr; ξ_m, ξ_{m+1} is the fraction of the initial diameter of the largest evaporating droplet of a polydisperse system at a given instant of time; ξ_i is the discrete initial diameter of the droplets in a polydisperse system, defined as a fraction of the diameter of the largest droplet; ξ_{im} is the discrete diameter of the evaporating droplet, expressed as a fraction of the initial diameter of the largest droplet at a given instant of time; τ_m is the droplet evaporation time from the beginning of the process to the instant at which the diameter becomes equal to $\xi_m d_{in}$, hr; τ_P is the evaporation time of a polydisperse system, hr; τ_M is the evaporation time of a monodisperse system, hr; $\Delta\Omega_{in}$, $\Delta\Omega_i$ are fractions of the total weight of the atomized liquid for droplets of diameters d_{in} and d_i, respectively; γ is the specific weight of the liquid, kg/m^3.

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22 April 1967

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